Analysis and compensation of dispersion-induced bit loss in a photonic A/D converter using time-wavelength interweaved sampling clock

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Abstract: In this paper, the timing jitter induced by the fiber dispersion in photonic A/D converters using time-wavelength interweaved sampling clocks generated by optical time-division-multiplexing (OTDM) with fiber delay lines is analyzed and effective bit loss is calculated. A compensation method is proposed to decrease the dispersion-induced jitter. Simulations are performed and the results show the validity of the proposed compensation method. An experimental demonstration is carried out to verify the theoretical expression derived.

OCIS codes: (230.0250) Optoelectronics; (060.2360) Fiber optics and optical communications: Fiber optics links and subsystems.

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1. Introduction

In order to generate time-wavelength interweaved sampling clock of tens even hundreds GHz for photonic analog-to-digital converter (ADC), a laser source with wide spectrum and ultra-short pulse output is needed [1–3]. Passive mode lock femto-second fiber laser is one of the appropriate candidates. The spectrum of a commercially available femto-second fiber laser may cover 50nm in the 1550 nm optical communications band. Such a spectrum can be sliced into 20-30 wavelengths. The wavelength-division-multiplexing (WDM) technique is desirable not only for increasing the pulse rate, but also for providing the possibility of parallel signal processing [1, 2]. However, the repetition rate of such lasers is usually very low. Therefore,

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optical time-division-multiplexer (OTDM) should be used to further aggrandize the pulse rate (i.e. the sampling rate). It is relatively convenient and cheap to use fiber delay lines (FDLs) to build the OTDM [2]. The drawback is that the fiber dispersion may aggravate timing jitter, which is one of the factors [4] that limit the effective bits of AD converters. In this paper, we analyze the effect of fiber dispersion with simulation verifications and propose a compensation scheme.

2. Theory

Figure 1 is the principle diagram that generates ultrahigh-speed time-wavelength interweaved sampling clock from a passive mode lock femto-second fiber laser (MLFL) with repetition rate of R_S. The WDM part has M wavelength channels, which are arranged at different time slot so as to form a pulse group by using M fiber delay lines. The OTDM part has N paths, which replicate the pulse group by N times. The repetition rate of the output pulse train is thus MNR_S.

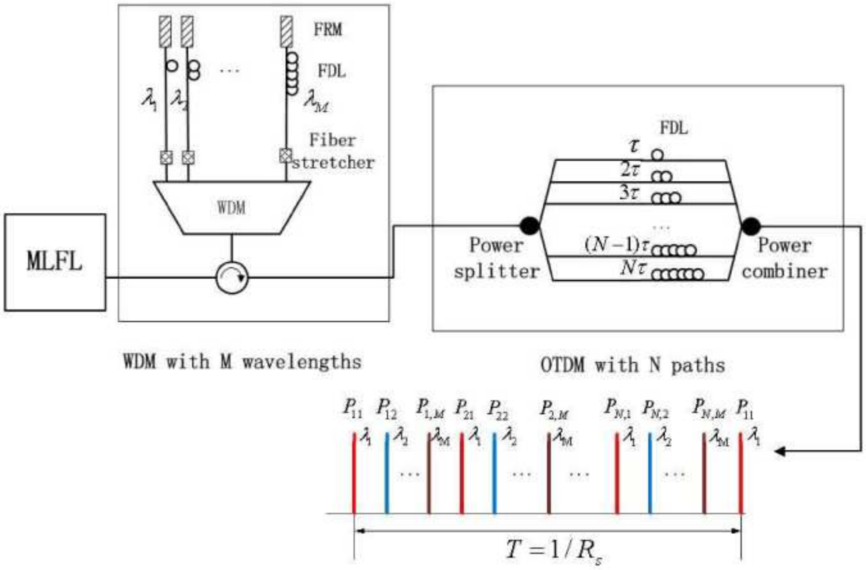

Fig. 1. The principle diagram generating time-wavelength interweaved sampling clock (FRM: Faraday Rotator Mirror)

In such a configuration, timing jitter, amplitude jitter and pulse width variation are three main factors that limit the effective bits [4, 5]. According to our experiment observation, both the WDM part and the OTDM part will induce power fluctuation and delay variation between different channels. In the WDM part, each channel uses a separated fiber. So the power fluctuation can be balanced through variable optical attenuator and the delay difference can be eliminated through careful adjustment. In the OTDM part, however, the fiber delay lines are shared by different channels. Though the amplitude fluctuation can be eliminated by using highly uniform couplers, the timing error caused by dispersion in the OTDM paths is hard to compensate.

Suppose the wavelength interval is $\Delta\lambda$ and the fiber length difference between the adjacent delay units is ΔL, then the jth wavelength and its delay, δ_j, in the WDM part, and the fiber length of ith delay unit, L_i, can be expressed as:

$$\lambda_j = \lambda_1 + (j-1)\Delta\lambda, 1 \le j \le M \qquad (1)$$

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$$\delta_j = \delta_1 + \frac{(j-1)}{MNR_s}, 1 \le j \le M \tag{2}$$

$$L_i = L_1 + (i-1)\cdot \Delta L, 1 \le i \le N \tag{3}$$

where λ_1 is the lowest wavelength, δ_1 is the shortest delay of the WDM, and L_1 is the length of the shortest path of the OTDM, respectively. The ideal delay caused by ΔL should be $1/NR_S$ and the ideal time delay for the wavelength j through path i should be:

$$\overline{t_{ij}} = \frac{i}{NR_s} + \delta_j, 1 \le i \le N, 1 \le j \le M \tag{4}$$

However, the group velocity, $v(\lambda_j)$, $j=1,2...M$, differs from each other due to fiber dispersion. The actual time delay of the wavelength j through path i is:

$$t_{ij} = \frac{L_i}{v(\lambda_j)} + \delta_j^{'}, 1 \le i \le N, 1 \le j \le M \tag{5}$$

where $\delta_j^{'}$ is the actual fiber delay of the jth channel. Suppose there is a wavelength, λ_s ($1 \le s \le M$), whose delay after transmission through the fiber section of ΔL is $1/NR_S$ (in fact, what we concern is the relative delay, so the assumption is valid which will be seen in Section 3). We regard this channel as the ideal channel and rewrite the jth wavelength using λ_s as:

$$\lambda_j = \lambda_s + (j-s)\Delta\lambda, 1 \le j \le M, 1 \le s \le M \tag{6}$$

Therefore the delay difference between λ_j and λ_s for the fiber section of ΔL is given by [6]:

$$\Delta\tau = \frac{\Delta L}{v(\lambda_j)} - \frac{\Delta L}{v(\lambda_s)} \approx \Delta L \cdot D(\lambda_{(j+s)/2}) \cdot \Delta\lambda (j-s) \tag{7}$$

where $D(\lambda_{(j+s)/2})$ is the dispersion at the center wavelength between λ_j and λ_s. So the delay deviation, i.e. the timing jitter, of the jth wavelength through ith path, τ_{ij}, is given by:

$$\begin{aligned} \tau_{ij} &= t_{ij} - \overline{t_{ij}} \approx L_i \cdot D(\lambda_{(j+s)/2}) \cdot \Delta\lambda (j-s) + (\delta_j^{'} - \delta_j) \\ &= [L_1 + (i-1)\Delta L] \cdot D(\lambda_{(j+s)/2}) \cdot \Delta\lambda (j-s) + (\delta_j^{'} - \delta_j), 1 \le i \le N, 1 \le j \le M \end{aligned} \tag{8}$$

Equation (8) shows the sampling time error of the pulse P_{ij} relative to the ideal case, which is caused by fiber dispersion in the WDM part and the OTDM part. As mentioned above, the timing error in the WDM part can be emendated by careful adjustment of the related fiber length. Therefore we will ignore the ($\delta_j - \delta_j^{'}$) term in the following analysis. According to the analysis by G. C. Valley, etc., the effective bits of photonic AD converters limited by the timing jitter is given by [4,5, 7]:

$$N_{eff} = \log_2 (1/\sqrt{3}\pi f \sigma_j) \tag{9}$$

where f is the highest frequency of the signal sampled [4]. In Eq. (9), σ_j is the RMS of the timing jitter. It contains both the laser source jitter and jitter induced by other factors such as dispersion-induced jitter given by Eq. (8). Taking into consideration the fact that τ_{ij} is periodic and can be equivalently treated as discrete uniform distribution, one can calculate the RMS of the dispersion induced timing jitter:

$$\sigma_j^d = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{M} |\tau_{ij}|^2}{MN}} \tag{10}$$

Since the laser source timing jitter and dispersion induced timing jitter are independent, one can get the total equivalent timing jitter as follows:

$$\sigma_j = \sqrt{(\sigma_j^d)^2 + (\sigma_j^s)^2} \tag{11}$$

where σ_j^s is the RMS timing jitter of the laser source.

3. Compensation method and analysis

We simulated the impact of dispersion induced timing jitter on the effective bits. The parameters used are as follows. The output pulse width of the MLFL is 70fs with optical spectrum of 1560±22.5nm, repetition period of 27.44ns and σ_j^s =150*fs*. The WDM part has 16 channels with channel spacing of 200GHz (1.6nm) and λ_1 =1540.56nm. The OTDM part has 64 paths with $\Delta L \approx$8.575cm and L_1 =5m. The highest frequency of the sampled microwave signal is *f*=10GHz. The dispersion coefficient, $D(\lambda)$, is given by [8]:

$$D(\lambda) = -\frac{2\pi c}{\lambda^2}\beta_2 \tag{12}$$

where *c* is the speed of light in vacuum, $\beta_2 \approx -20\, ps^2 / km$ in the 1550nm range.

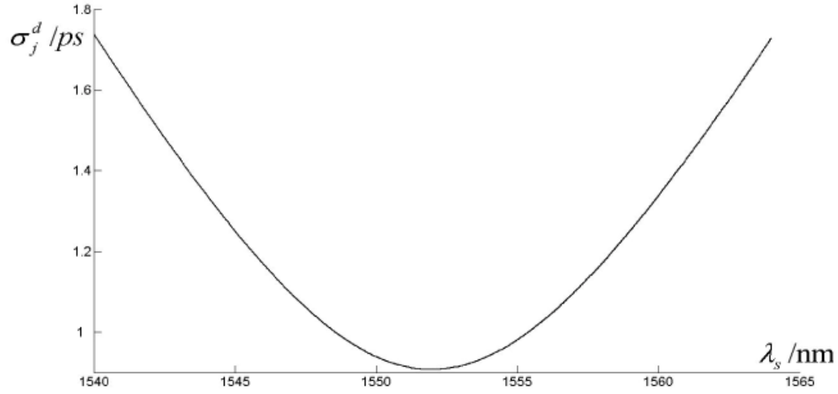

Fig. 2. Dispersion-induced timing jitter

First we calculated the equivalent timing jitter σ_j^d caused by dispersion for different λ_s as shown in Fig. 2. Obviously, the dispersion induced timing jitter is the lowest when *s=M/2*. It means that when we make an OTDM, the "ideal wavelength" should be the central wavelength of the spectrum used. Even though, the jitter caused by dispersion is still large and can cause significant effective bit loss. So it must be compensated. It is hard to compensate the dispersion for each wavelength in OTDM because OTDM paths are shared. One method to alleviate the dispersion effect is to use low dispersion fiber, such as dispersion shifted fiber (DSF) or dispersion flattened fiber, provided that devices such as optical couplers based on such fiber with precise coupling ration and ultra flatness over the interested bandwidth are available. The other way to decrease the jitter is to compensate the dispersion in the WDM part.

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Set $s=M/2$, then stretch the jth ($j=1,2,\ldots,M$) fiber in WDM part so that it produces an extra compensation C_j^t given by:

$$C_j^t = C - C_j \tag{13}$$

where C is a constant. The relative delay to the "ideal wavelength channel", λ_s, may be positive or negative. It is more convenient to produce extra delay by stretching the fiber. C is used to avoid the case that the total extra delay, C_j^t, is negative. Because C is effectively equal to each other, we will not include it in the equations of the following analysis. The term "effectively equal" indicates that the dispersion induced delay for each wavelength, i.e., ($\delta_j - \delta_j^{'}$), is considered in C. After compensation, the sampling error becomes:

$$\tau_{ij}' = L_i \cdot D(\lambda_{j/2+M/4}) \cdot \Delta\lambda(j - \frac{M}{2}) - C_j \tag{14}$$

Different choice of C_j adds different effects on σ_j^d. In this paper we use a simple strategy of compensation, i.e., C_j is given by:

$$C_j = L_k \cdot D(\lambda_{j/2+M/4}) \cdot \Delta\lambda(j - \frac{M}{2}), 1 \le k \le N \tag{15}$$

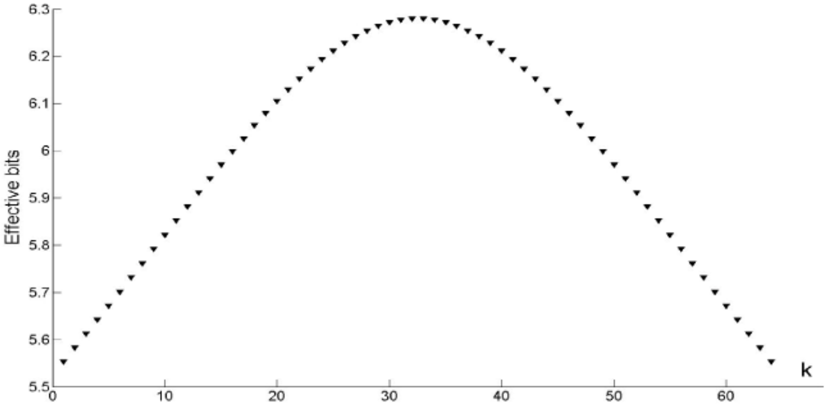

Fig. 3. Relationship between the effective bits and k

Hence the setting of C_j becomes the problem of choosing optimal k. Figure 3 shows the relationship between the effective bits and k when $s=M/2$. We can find that when $k=N/2$, the impact of the timing jitter is the lowest. Therefore, Eq. (15) gives the amount of compensation of the jth wavelength in the WDM part for the compensation strategy mentioned above, where L_k is given by Eq. (3) with $k=N/2$.

In order to prove the validity of the compensation scheme, we simulated the effective bits for different number of wavelengths. Figure 4 shows the relationship for the parameters given above. It can be clearly seen that the fiber dispersion induced timing jitter has strong influence on the effective bits in the system employing fiber delay lines to generate time-wavelength interweaved sampling clocks as shown in Fig. 1, especially when more wavelengths are used for generating ultrahigh rate sampling clocks. The simple compensation scheme we proposed can improve the performance of the photonic ADC by increasing 2-3 effective bits. We also simulated the extreme cases that the laser source is very stable (σ_j^s =10*fs*) and very noisy (σ_j^s =300*fs*). The results still show effectiveness of the compensation scheme.

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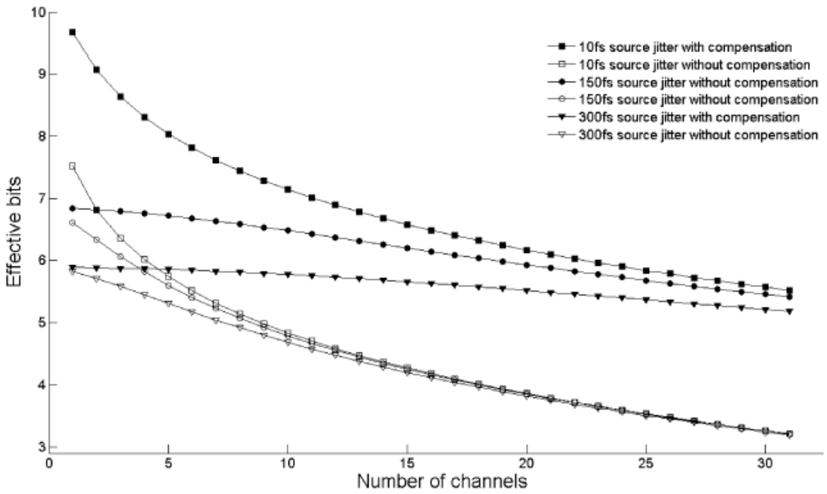


Fig. 4. Relationship between effective bits and number of channels

Figure 5 shows the effective bits versus channel number when DSF is used to implement the OTDM. The DSF dispersion is modeled by formulas $D(\lambda) = (\lambda - \lambda_0)S_0$ recommended by EIA and ITU-T [6], where λ_0 is the zero dispersion wavelength around 1550 nm and S_0 is the dispersion slope. It can be seen that DSF can provide higher effective bits in comparison with the conventional single mode fiber. However, the performance can still be further improved by employing the compensation method proposed, especially when a stable laser source and more channels are used.

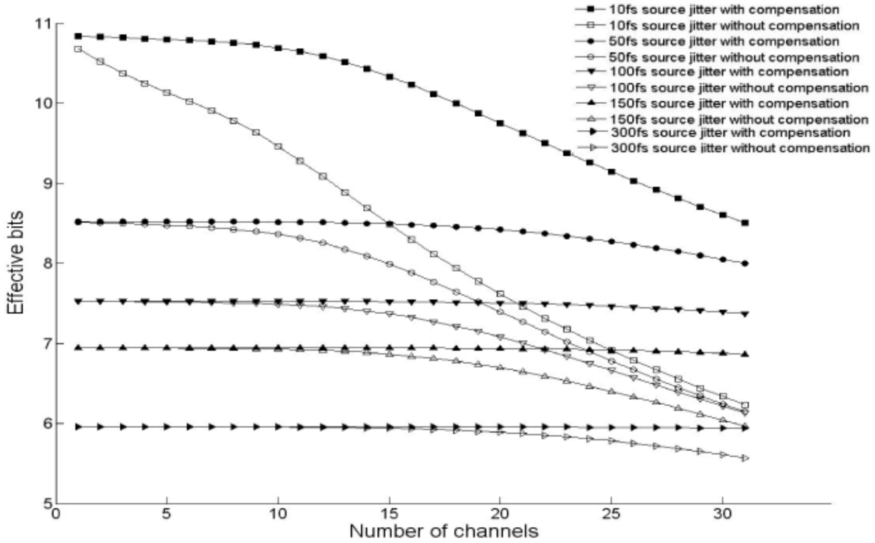
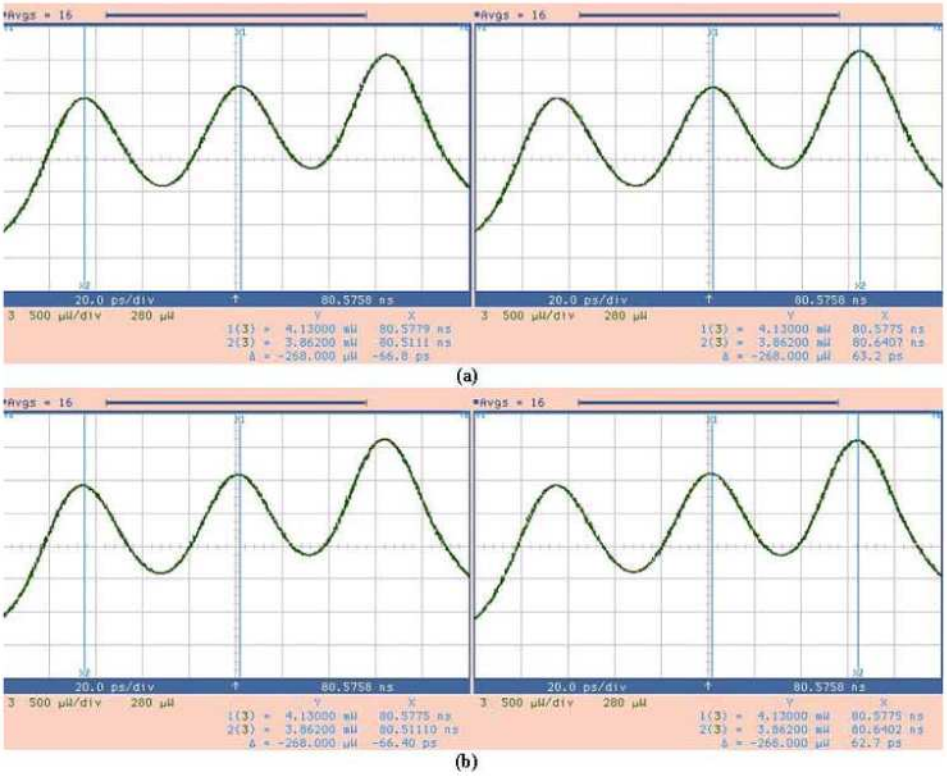


Fig. 5. Relationship between effective bits and number of channels for DSF-based OTDM

In addition to the theoretical calculation, the right C_j value can be found experimentally as follows. Firstly, measure the time interval between the input pulses of *j*th wavelength and the reference wavelength. Then measure the time interval again after the pulse train passes through the *N/2*th path of the OTDM as shown in Fig. 6 (a). C_j can be obtained by calculating the difference of the two time intervals.

Figure 6 shows the measured results. In our experiment, the reference pulse is with wavelength of 1554.13nm and the two pulses used as example are at 1546.12nm and 1565.5 nm. Their intervals with the reference pulse before input into the OTDM are 66.4*ps* and 62.7*ps*, respectively. After passing through the OTDM, these values changed to 66.8*ps* and 63.2*ps*, respectively, as shown in Fig. 6(a). Hence C values are equal to −0.4*ps* and 0.5*ps*, respectively. From Eq. (15), one can calculate the theoretic values are −0.36*ps* and 0.51*ps*, respectively, which are in good agreement with the test result.

The compensation can be implemented by high precise fiber stretcher [9, 10] or fused micro drawing [11]. Figure 6(b) is the measured results after compensation using manually tunable mechanical stretchers we designed and implemented. It can work stably, as shown in Fig. 7, with resolution sufficient for such applications.



(a)

(b)

Fig. 6. Measurement of pulse intervals to determine C_j. (a) Output from the N/2th path of the OTDM before compensation. (b) Output from the *N/*2th path of the OTDM after compensation

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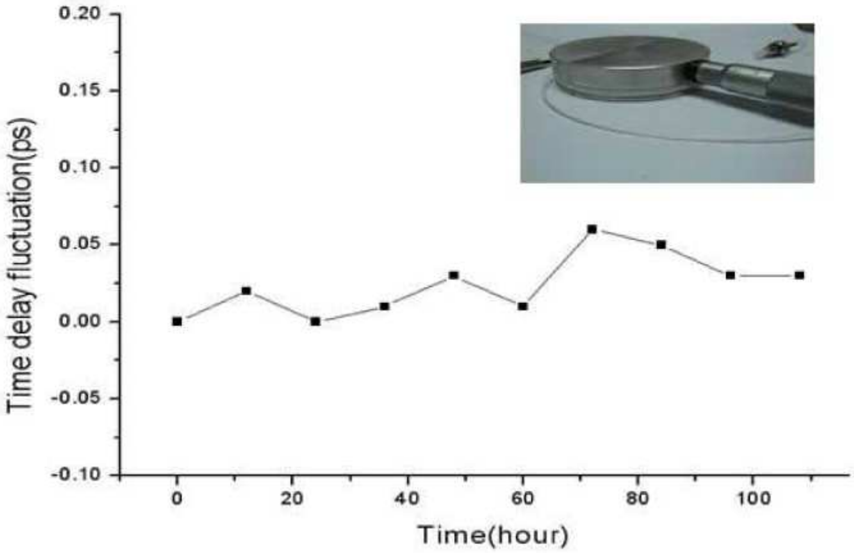

Fig. 7. The measured stability of the designed mechanical fiber stretcher

4. Conclusion

In this paper, the timing jitter caused by dispersion in the shared OTDM fiber channels in a photonic A/D converter is analyzed. Expressions to calculate the equivalent RMS timing jitter are presented. A compensation strategy is proposed by simply stretching the fiber in the WDM part. The simulation results show that, by using the strategy, the performance of the photonic A/D converter can be effectively improved by 2-3 bits. Simulation shows that it is effective even if the OTDM is configured with dispersion shifted fiber. Experimental measurement verified the validity of the theoretical expressions derived.

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